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# Retardation effects in the rotating string model

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**Abstract.** A new method to study the retardation effects in mesons is presented. It is based on a generalized rotating string model, in which a nonzero value of the relative time between the quark and the antiquark is allowed. This approach leads to a retardation term in the Hamiltonian which behaves as a perturbation of the nonretarded Hamiltonian and preserves the Regge trajectories for light mesons. The straight-line ansatz is used to describe the string, and the relevance of this approximation is tested. It is shown that the string is actually curved because of retardation, but this bending does not bring a relevant contribution to the energy spectrum of the model.

**PACS.** 12.39.Ki Relativistic quark model – 12.39.Pn Potential models – 14.40.-n Mesons

## 1 Introduction

The retardation effect between two interacting particles is a relativistic phenomenon, due to the finiteness of the interaction speed. Light mesons are typical systems in which these effects can significantly contribute to the dynamics, since the light quarks can move at a speed close to the speed of light. We present here a generalization of the rotating string model (RSM) [1,2] developed in refs. [3,4], which aims to take into account these effects in mesons.

## 2 Rotating string model with a nonzero relative time

The RSM is an effective model derived from the QCD Lagrangian, describing a meson by a quark and an antiquark linked by a straight string. Both particles are considered as spinless because spin interactions are sufficiently small to be added in perturbation. Our method to treat the retardation effects must be considered as a first trial to take into account such contributions in mesons. It relies on the hypothesis that the relative time between the quark and the antiquark must have a nonzero value. Consequently, in our approach, the evolution parameter of the system is not the common proper time of the quark, the antiquark and the string, but the time coordinate of the centre of mass which plays the role of an "average" time. The RSM with a nonzero relative time has been studied in detail in ref. [3]. So, we simply recall here the main points of this work.

Starting from the QCD Lagrangian and particularising it to the case of an interacting quark-antiquark pair, an effective Lagrangian can be derived. In units where $\hbar = c = 1$, it reads [1]

$$\mathcal{L} = -m_1\sqrt{\dot{\boldsymbol{x}}_1^2} - m_2\sqrt{\dot{\boldsymbol{x}}_2^2} -a\int_0^1 \mathrm{d}\theta\sqrt{(\dot{\boldsymbol{w}}\boldsymbol{w}')^2 - \dot{\boldsymbol{w}}^2\boldsymbol{w}'^2}. \quad (1)$$

The first two terms are the kinetic energy operators of the quark and the antiquark, whose current masses are $m_i$. These two particles are attached by a Nambu-Goto string with a tension $a$. $\boldsymbol{x}_i$ and $\boldsymbol{w}$ are the coordinates of the quark $i$ and of the string, respectively. We defined $\dot{\boldsymbol{x}}_i = \partial_\tau \boldsymbol{x}_i$, $\dot{\boldsymbol{w}} = \partial_\tau \boldsymbol{w}$, and $\boldsymbol{w}' = \partial_\theta \boldsymbol{w}$. The string is generally assumed to be a straight line linking the quark to the antiquark, that is

$$\boldsymbol{w} = \theta\boldsymbol{x}_1 + (1-\theta)\boldsymbol{x}_2, \quad \theta \in [0,1]. \quad (2)$$

Such an ansatz is suggested in particular by lattice QCD calculations, which show that the chromoelectric field between the quark and the antiquark appears to be roughly constant on a straight line joining the two particles [5].

Starting from Lagrangian (1) with a straight string given by eq. (2), one usually makes the equal-time ansatz, *i.e.*

$$x_1^0 = x_2^0 = w^0 = \tau = \bar{t}. \quad (3)$$

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Then, we have $\boldsymbol{r} = (0, \boldsymbol{r})$, and $\boldsymbol{R} = (t, \boldsymbol{R})$. This procedure considerably simplifies the equations, but neglects the relativistic retardation effects due to a possible nonzero value of the relative time $\sigma$. That is why we made in ref. [3] a less restrictive hypothesis: We identified the temporal coordinate of the centre of mass with the evolution parameter, $\bar{t} = \tau$, and we allowed a nonvanishing relative time $\sigma$.

It is then possible to derive from the Lagrangian (1) a set of three equations for the RSM with a nonzero relative time,

$$0 = \mu_1 y_1 - \mu_2 y_2 - \frac{ar}{y_t}\left(\sqrt{1-y_1^2} - \sqrt{1-y_2^2}\right), \tag{4a}$$

$$\frac{L}{r} = \frac{1}{y_t}(\mu_1 y_1^2 + \mu_2 y_2^2) + \frac{ar}{y_t^2}(F(y_1) + F(y_2)), \tag{4b}$$

$$H = \frac{1}{2}\left[\frac{p_r^2 + m_1^2}{\mu_1} + \frac{p_r^2 + m_2^2}{\mu_2} + \mu_1(1+y_1^2) + \mu_2(1+y_2^2)\right] + \frac{ar}{y_t}(\arcsin y_1 + \arcsin y_2) + \Delta H, \tag{4c}$$

with

$$F(y_i) = \frac{1}{2}\left[\arcsin y_i - y_i\sqrt{1-y_i^2}\right]. \tag{5}$$

$p_r$ is the radial momentum and $y_i$ is the transverse velocity of the quark $i$, and $y_t = y_1 + y_2$. The first relation gives the cancellation of the total momentum in the centre-of-mass frame, while the two last ones define, respectively, the angular momentum and the Hamiltonian of the system. Two auxiliary fields, denoted as $\mu_i$, are introduced to simplify the computation. They can however be interpreted as dynamical masses of the quarks whose current masses are $m_i$ [6]. They can be eliminated by minimising the energies with respect to them.

Equations (4) are identical to those of the usual RSM (see, for example, ref. [6]), but a perturbation of the Hamiltonian, denoted $\Delta H$, is now present. It contains the contribution of the retardation effects and is given by [3]

$$\Delta H = -\frac{\Sigma^2}{2a_3} + \frac{c_2}{a_3}\Sigma\sigma - c_1\sigma - \frac{c_2^2}{2a_3}\sigma^2 + \frac{1}{2}a_4\sigma^2, \tag{6}$$

where $\Sigma$ is the canonical momentum associated with the relative time $\sigma$. $c_2$, $a_3$, and $a_4$ are complicated functions of the spatial variables [6].

Let us now consider the quantized version of our model: $L \to \sqrt{\ell(\ell+1)}$, $[r, p_r] = i$, $[\sigma, \Sigma] = -i$. The Hamiltonian (4c) has then the following structure:

$$H(\sigma, r) = H_0(r) + \Delta H(\sigma, r). \tag{7}$$

The relative time only appears in the perturbation, and $H_0$ depends only on the radial variables. So, we can assume that the total wave function reads

$$|\psi(\boldsymbol{r})\rangle = |A(\sigma)\rangle \otimes |R(r)\rangle \otimes |Y_{\ell\, m}(\theta,\phi)\rangle\,, \tag{8}$$

where $|R(r)\rangle$ is a solution of the eigenequation

$$H_0(r)\,|R(r)\rangle = M_0\,|R(r)\rangle\,. \tag{9}$$

Such a problem can be numerically solved, for instance, by the Lagrange mesh technique [7]. The total mass is thus given by

$$M = M_0 + \langle A(\sigma)| \otimes \langle R(r)|\, \Delta H(r,\sigma)\, |R(r)\rangle \otimes |A(\sigma)\rangle = M_0 + \Delta M. \tag{10}$$

As an excited state with respect to the relative time is irrelevant, the contribution $\Delta M$ is then given by the fundamental state of the eigenequation

$$\Delta\mathcal{H}(\sigma)\,|A(\sigma)\rangle = \Delta M\,|A(\sigma)\rangle\,, \tag{11}$$

where

$$\Delta\mathcal{H}(\sigma) = \langle R(r)|\,\Delta H(r,\sigma)\,|R(r)\rangle\,. = -\frac{1}{2\langle a_3\rangle}\left[\Sigma^2 + \langle c_2^2 - a_4 a_3\rangle\sigma^2\right] \tag{12}$$

in the case $m_1 = m_2$. This constraint eliminates the unphysical degree of freedom due to the introduction of the relative time. Let us mention two consequences of eq. (11).

Firstly, the retardation contribution $\Delta M$ is negative,

$$\Delta M = -\frac{1}{2}\sqrt{\langle c_2^2 - a_4 a_3\rangle / \langle a_3\rangle^2}, \tag{13}$$

and it thus decreases the meson mass. It has been shown for a long time that quark models were able to roughly fit the hadron spectrum [8]. However, an arbitrary negative constant is still needed to shift the energies to the correct values. Our retardation term could thus be a good physical candidate to replace this arbitrary constant.

Secondly, the temporal part of the wave function reads

$$A(\sigma) = \left(\frac{\beta}{\pi}\right)^{1/4} \exp\left(-\frac{\beta}{2}\sigma^2\right), \tag{14}$$

with

$$\beta = \sqrt{\langle c_2^2 - a_4 a_3\rangle}. \tag{15}$$

It is a Gaussian function centered around $\sigma = 0$. This provides an interpretation of the equal-time ansatz (3) as the most probable configuration of the system.

A simple calculation shows that $\Delta M$ preserves the Regge trajectories of light mesons [3]. Moreover, our model, supplemented by a one-gluon-exchange potential and quark self-energy term can rather well reproduce the spin averaged experimental meson masses of light and heavy mesons.

## 3 String shape beyond the straight-line ansatz

It is worth noting that the use of a nonvanishing relative time is not really compatible with the straight-line ansatz. This can be seen by the following simple considerations: Let us assume that the world sheet of the system in the centre-of-mass frame is a helicoid area in the case of exactly circular quark orbits. The shape of the string is then

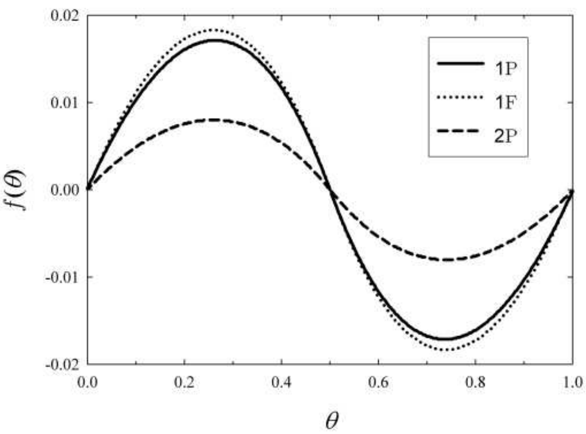


**Fig. 1.** The curved string linking the quark to the antiquark for different states.

a straight line for a slice at constant time and a curve for a slice not at constant time. This problem was investigated in ref. [4], whose main points are summed up in this section.

As the meson evolves in a plane, we can use for the string the complex coordinates $(w^0, w, w^*)$ defined by

$$w = \frac{1}{\sqrt{2}}(w^1 + iw^2). \tag{16}$$

With these coordinates, a curved string rotating at a constant angular speed $\omega$ can be described by the ansatz

$$w^0(\theta, \tau) = \tau + \theta\,\sigma(\tau)/2, \tag{17a}$$

$$w(\theta, \tau) = \frac{r(\tau)}{2\sqrt{2}}\,[\theta + if(\theta)]\exp[i\omega\tau], \tag{17b}$$

where the spatial deformation $f$ has been introduced to take into account the relative time $\sigma$. Equations (17) clearly describe a curved string, as can be seen by rewriting it when $\tau = 0$. Then we have simply $w^1 \propto \theta$ and $w^2 \propto f(\theta)$. As $\sigma$ is assumed to be arbitrary, only $f$ has to be determined thanks to the equations of motion of the Nambu-Goto Lagrangian, supplemented by the requirement that $f$ vanishes at the centre of mass and at the ends of the string. The trivial solution $f = 0$ is only valid if $\sigma = 0$. This is the straight line without retardation. We already gave in ref. [3] arguments showing that the bending was small. Consequently, we can linearize the equations in $f$ and solve them numerically. The conclusion of ref. [4] is that the string is curved, with a state-dependent amplitude. The bending is maximal for the $1F$ state and decreases with the quantum numbers, as illustrated in fig. 1. Moreover, the string shape between the centre of mass and one quark can be roughly approximated by the expression

$$f(\rho) \approx -\frac{\sigma\omega}{2}\,\rho\,(1-\rho), \tag{18a}$$

with $\rho \in [0,1]$, $\rho = 0$ being the centre of mass and $\rho = 1$ the quark. The other part of the string is readily obtained by a central symmetry with respect to the centre of mass. It is worth mentioning that in the case of a vanishing angular momentum, $\omega = 0$, the solution is trivially $f(\rho) = 0$. Even when the retardation is included, the string is straight when the angular momentum is zero.

Since the string brings an energetic contribution to the meson which is proportional to its length, it is interesting to evaluate the ratio between the lengths of both the curved and the straight strings. It is given by [4]

$$\Delta L/L \leq 2 \times 10^{-3}. \tag{19}$$

The length of the string is only modified by some tenths of percent, because of the bending induced by retardation effects. As the typical mass scale for mesons is around 1–2 GeV, the correction due to the curved string is around 1–4 MeV. Such an order of magnitude was also obtained in a previous study of the string deformation due to nonuniform rotation [9]. The contribution of the bending of the string to the mass spectrum seems thus very small. This is also small compared with the retardation contribution, which can be around 100 MeV for massless quarks.

## 4 Conclusion

By allowing the relative time between the quark and the antiquark in a meson to be nonzero, we have been able to obtain a generalized rotating string model in which the retardation effects contribute as a perturbation of the non-retarded Hamiltonian. This contribution does not destroy the Regge trajectories and can lead to a good agreement with the experimental meson spectrum. We also showed that the string was curved due to retardation, but this bending does not influence significantly the mass spectrum of the model. In conclusion, our study reveals that the usually neglected retardation effects could be a relevant physical mechanism, in particular in the light meson sector where the retardation can give a contribution around 100 MeV. It should be interesting to study the dependence in the relative time of other observables, like the decay width of mesons, for example, in order to make further comparisons with experiment. We leave this for future investigations.

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